PLANE CONTACT PROBLEMS OF THE THEORY OF ELASTICITY FOR NONCLASSICAL REGIONS IN THE PRESENCE OF WEAR

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In this paper an effective method is proposed for the solution of plane contact problems for nonclassical regions [1] in the presence of wear. The inertia forces arising from the motion of the punch [2, 3] are not taken into account.

1. Statement of the Problem. It has been established experimentally [4, 5] that the rate of wear is a function of shear forces and the averaged modulus of the velocity of sliding, where for abrasive wear we use, as a rule, the linear relation

$$w = k_1 V\tau(x,\ t), \tag{1.1}$$

where k_1 is the coefficient of proportionality between the work of friction forces and the amount of material removed. Hence it follows that the displacement of the punch in the direction perpendicular to the surface of the region, as a result of its wear, has the form

$$v_* = k_1 V \int_0^t \tau(x,t)\,dt = \varkappa \int_0^t q(x,t)\,dt \quad (\varkappa = k_1 k_2 V), \tag{1.2}$$

where k_2 is the coefficient of friction; q(x, t) is the contact pressure. For normal displacement of a punch of width $2a$, as a result of the elastic deformation of the region, we have the expression [1]

$$v = \frac{1}{\pi\theta}\int_{-a}^{a} q(\xi,t) K\left(\frac{\xi - x}{\mu}\right) d\xi \quad (|x| \leqslant a); \tag{1.3}$$

$$K(y) = \int_0^\infty \frac{L(u)}{u}\cos uy\,du \quad \left(y = \frac{\xi - x}{\mu}\right), \tag{1.4}$$

where θ is a certain combination of elastic constants determined by the concrete problems. We assume that:

1) the function $L(u)u^{-1}$ is continuous, real, and even on the real axis;

2) the function

$$L(u)u^{-1} > 0 \quad (|u| < \infty); \tag{1.5}$$

3) the function

$$L(u) = Au + O(u^2) \ (u \to 0), \quad \frac{L(u)}{u} = C^2 u^{-2p}[1 + O(u^{-s})] \ (u \to \infty), \quad 0.25 < p < 1, \tag{1.6}$$

A, C, p, s are constants, with $s > p$ for $p \geqslant 0.5$, $s > 1 - p$ for $p < 0.5$.

The condition of contact of the punch with the region obviously has the form

$$v + v_* = \gamma(t) + \beta(t)x - f(x) \ (|x| \leqslant a). \tag{1.7}$$

Here $\gamma(t) + \beta(t)x$ is the rigid-body motion of the punch under the action of the force P(t) and the moment M(t) applied to it; f(x) is the function describing the form of the base of the punch.

Substituting (1.2), (1.3) into (1.7), we obtain the integral equation for the determination of the unknown contact stresses

$$\int_{-a}^{a} q(\xi,t) K\left(\frac{\xi - x}{\mu}\right) d\xi = \pi\theta[\gamma(t) + \beta(t)x - f(x)] - \pi\varkappa\theta \int_0^t q(x,t)\,dt \tag{1.8}$$

$$(|x| \leqslant a);$$

$$P(t) = \int_{-a} q(\xi,t)\,d\xi, \quad M(t) = \int_{-a}^{a} \xi q(\xi,t)\,d\xi. \tag{1.9}$$

Going in (1.8), (1.9) to dimensionless variables and notation according to the expressions

$$\xi'a = \xi,\ x'a = x,\ \lambda = \mu/a,\ t = at'/\pi\varkappa\theta,\ \gamma(t) = a\gamma'(t'),\ \beta(t) = \beta'(t'),$$
$$f(x) = af'(x'),\ q(\xi,\ t) = \theta\varphi(\xi',\ t'),\ P(t) = aP'(t'),\ M(t) = a^2M'(t')$$

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(primes will be omitted in the following), we obtain the integral equation of the plane contact problem with wear

$$\int_{-1}^{1} \varphi(\xi, t) K\left(\frac{\xi - x}{\lambda}\right) d\xi = \pi [\gamma(t) + \beta(t) x - f(x)] - \int_{0}^{t} \varphi(x, t)\, dt \quad (|x| \leqslant 1); \tag{1.10}$$

$$P(t) = \int_{-1}^{1} \varphi(x, t)\, dx, \quad M(t) = \int_{-1}^{1} x\varphi(x, t)\, dx. \tag{1.11}$$

Here and subsequently we assume $0 \leqslant t \leqslant T < \infty$, where the quantity T is sufficiently large but such that $\gamma(t)$ and $\beta(t)$ have the order of displacements in the linear theory of elasticity.

We note that for t = 0 the integral equation (1.10) assumes the form, known from the theory of static contact problems [1],

$$\int_{-1}^{1} \varphi(\xi, 0) K\left(\frac{\xi - x}{\lambda}\right) d\xi = \pi [\gamma(0) + \beta(0) x - f(x)] \quad (|x| \leqslant 1). \tag{1.12}$$

Before proceeding to the solution of (1.10), we establish properties of its kernel which are important in the following.

The following lemma is proved in [6].

LEMMA. In the case $y = (\xi - x)\lambda^{-1} \to 0$ the following estimates are valid:

$$K(y) = O(|y|^{2p-1}),\ p < 0.5;\ K(y) = O(\ln|y|),\ p = 0.5;$$
$$K(y) = O(1),\ p > 0.5.$$

For $|y| > \varepsilon > 0$ the function K(y) is continuous and vanishes for $|y| \to \infty$.

With the inequality

$$\int_{-1}^{1}\int_{-1}^{1} \left[K\left(\frac{\xi - x}{\lambda}\right)\right]^2 d\xi dx = N^2 < \infty \quad (N = \text{const}),$$

arising from the lemma, taken into account, we have the following theorem.

THEOREM 1. The operator

$$B\varphi = \int_{-1}^{1} \varphi(\xi) K\left(\frac{\xi - x}{\lambda}\right) d\xi \quad (|x| \leqslant 1,\ 0 < \lambda < \infty)$$

acts from $L_2(-1, 1)$ into $L_2(-1, 1)$ completely continuously.

Here $L_2(-1, 1)$ is a space of functions summable on the segment [−1, 1] with square.

We also note that the operator $B\varphi$, in view of the representation (1.4), is self-adjoint. Therefore according to the general theory [7] of self-adjoint, completely continuous operators in the Hilbert space, it has a countable set of nontrivial even eigenvalues $\varphi_{2k}(x)$ $(k \geqslant 1)$ with eigenvalues α_{2k}, and a countable set of nontrivial odd eigenfunctions $\varphi_{2k+1}(x)(k \geqslant 1)$ with eigenvalues α_{2k+1}. At the same time all α_j are real and $|\alpha_1| \leqslant |\alpha_2| \leqslant \ldots \leqslant |\alpha_n| < \infty$, $\lim_{j\to\infty} |\alpha_j| = \infty$.

In addition, with the expressions (1.4), (1.5) and results of [6] taken into account, we can easily prove the following theorem.

THEOREM 2. The operator B_φ is positive definite in $L_2(-1, 1)$. From the latter it follows that all $\alpha_j > 0$ $(j \geqslant 1)$ and the system of eigenfunctions of the operator $B\varphi$ is complete in $L_2(-1, 1)$.

2. The Case of Linear Variation with Time of the Rigid Displacement of the Punch. Let the rigid displacement of the punch vary with time according to the law

$$\gamma(t) = \gamma_0 + \gamma_1 t,\ \beta(t) = \beta_0 + \beta_1 t.$$

Side by side with Eq. (1.10) we consider

$$\int_{-1}^{1} [\varphi(\xi, t) - \varphi(\xi, 0)] K\left(\frac{\xi - x}{\lambda}\right) d\xi = \pi [\gamma_1 t + \beta_1 t x] - \int_{0}^{t} \varphi(x, t)\, dt \quad (|x| \leqslant 1,\ 0 \leqslant t \leqslant T) \tag{2.1}$$

and will seek its solution in the form

$$\varphi(x, t) = \varphi_0(x) + \varphi_1(x) + \sum_{k=1}^{\infty} [c_{2k}(t)\varphi_{2k}(x) + c_{2k+1}(t)\varphi_{2k+1}(x)]; \tag{2.2}$$

$$\alpha_{2k}\int_{-1}^{1}\varphi_{2k}(\xi)K\left(\frac{\xi-x}{\lambda}\right)d\xi=\varphi_{2k}(x)\quad(|x|\leqslant 1,\ k\geqslant 1); \tag{2.3}$$

$$\alpha_{2k+1}\int_{-1}^{1}\varphi_{2k+1}(\xi)K\left(\frac{\xi-x}{\lambda}\right)d\xi=\varphi_{2k+1}(x)\quad(|x|\leqslant 1,\ k\geqslant 1). \tag{2.4}$$

Substituting (2.2) into (2.1), assuming

$$\varphi_0(x)=\pi\gamma_1,\ \varphi_1(x)=\pi\beta_1 x \tag{2.5}$$

and equating in the resulting relation the coefficients of the left and right sides at eigenfunctions of the operator $B\varphi$ of the same number, we obtain

$$\alpha_{2k}\int_{0}^{t}c_{2k}(\tau)\,d\tau+c_{2k}(t)=c_{2k}(0); \tag{2.6}$$

$$\alpha_{2k+1}\int_{0}^{t}c_{2k+1}(\tau)\,d\tau+c_{2k+1}(t)=c_{2k+1}(0)\quad(k\geqslant 1,\ 0\leqslant t\leqslant T). \tag{2.7}$$

Solving (2.6), (2.7), we find

$$c_{2k}(t)=c_{2k}(0)\,e^{-\alpha_{2k}t},\quad c_{2k+1}(t)=c_{2k+1}(0)\,e^{-\alpha_{2k+1}t}. \tag{2.8}$$

We consider next the even case, when f(x) is an even function, $\beta(t)\equiv 0$, bearing in mind that for the odd case we can proceed completely analogously.

We shall seek $\varphi_{2k}(x)$ in (2.3) in the form

$$\varphi_{2k}(x)=\sum_{m=0}^{\infty}a_m^{(k)}P_{2m}^{*}(x), \tag{2.9}$$

where $\{P_n^{*}(x)\}$ is a system of normed Legendre polynomials [7] closed in $L_2(-1,1)$.

Expanding the function K(y) in a dual series of these polynomials

$$K(y)=\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}e_{ij}(\lambda)P_{2i}^{*}(\xi)P_{2j}^{*}(x) \tag{2.10}$$

and using the integral [8]

$$\int_{0}^{1}P_{2n}(x)\cos ux\,dx=(-1)^n\sqrt{\frac{\pi}{2u}}J_{\frac{1}{2}+2n}(u),$$

we represent the coefficients of the expansion in (2.10) in the form

$$e_{ij}(\lambda)=(-1)^{i+j}\pi\lambda\sqrt{4i+1}\sqrt{4j+1}\int_{0}^{\infty}\frac{L(u)}{u^2}J_{\frac{1}{2}+2i}\left(\frac{u}{\lambda}\right)J_{\frac{1}{2}+2j}\left(\frac{u}{\lambda}\right)du.$$

Substituting (2.9), (2.10) into (2.3), using the orthogonality property of Legendre polynomials and equating the coefficients of the right and left sides of the resulting relation at polynomials of the same number, we have

$$\alpha_{2k}\sum_{m=0}^{\infty}a_m^{(k)}e_{jm}(\lambda)=a_j^{(k)}\quad(j=0,1,2,\ldots). \tag{2.11}$$

Using the results of the lemma and Theorem 1, we can show that the operator standing on the left side (2.11) acts in the space l_2 and is there completely continuous for $\lambda\in(0,\ \infty)$. Here l_2 is a complete space of quadratically summable sequences.

Thus, to the infinite system (2.11) the theorem of Hilbert [9] about its solvability is applicable. For there to exist a nontrivial solution of the system, we equate its determinant to zero; we obtain an equation for finding a countable set of eigenvalues α_{2k}. Having determined α_{2k}, we find then $a_m^{(k)}$, having expressed it in terms of $a_0^{(k)}$,

$$a_m^{(k)}=a_0^{(k)}b_m^{(k)}\quad(b_0^{(k)}=1). \tag{2.12}$$

As a result we have

$$\varphi_{2k}(x) = a_0^{(k)}\psi_{2k}(x), \quad \psi_{2k}(x) = \sum_{m=0}^{\infty} b_m^{(k)} P_{2m}^{*}(x) \quad (k \geqslant 1). \tag{2.13}$$

We now choose the constants $a_0^{(k)}$ in (2.12), (2.13) from the norming condition of the eigenfunctions $\varphi_{2k}(x)$ of the operator $B\varphi$, i.e.,

$$\int_{-1}^{1} \varphi_{2k}(x)\varphi_{2n}(x)\,dx = a_0^{(k)}a_0^{(n)} \sum_{m=0}^{\infty} b_m^{(k)} b_m^{(n)} = \delta_{kn} \\ (k, n = 1, 2, 3, \ldots) \tag{2.14}$$

(δ_{kn} is the Kronecker symbol).

After finding $a_0^{(k)}$ from the system (2.14) we determine the sought eigenfunctions of the operator $B\varphi$. We now satisfy the integral equation (1.12) by appropriate choice of a countable set of constants $c_{2k}(0)(k \geqslant 1)$. We assume that $f(x) \in L_2(-1,1)$, and expand it in a Fourier series of eigenfunctions of the operator $B\varphi$

$$f(x) = \sum_{i=0}^{\infty} f_i \varphi_{2i}(x). \tag{2.15}$$

Taking into account (2.2), (2.8) and the relation

$$1 = \sqrt{2}\sum_{k=1}^{\infty} a_0^{(k)}\varphi_{2k}(x),$$

we obtain

$$\varphi(x, 0) = \sum_{k=1}^{\infty} \left[\sqrt{2}\,\pi\gamma_1 a_0^{(k)} + c_{2k}(0)\right]\varphi_{2k}(x). \tag{2.16}$$

Substituting (2.15), (2.16) into (1.12), using the orthogonality property of the functions $\varphi_{2k}(x)$ and equating in the resulting expression the coefficients of the right and left sides at functions of the same number, we obtain

$$c_{2k}(0) = \pi\sqrt{2}\,a_0^{(k)}(\alpha_{2k}\gamma_0 - \gamma_1) - \pi f_k \alpha_{2k} \quad (k \geqslant 1). \tag{2.17}$$

After determining $c_{2k}(0)$ from (2.17) we construct the formal solution of the problem according to the expression (2.2). At the same time from the expression (1.11) we find the force acting on the punch

$$P(t) = 2\pi\gamma_1 + \sum_{k=1}^{\infty} P_k e^{-\alpha_{2k}t}, \\ P_k = c_{2k}(0)\int_{-1}^{1} \varphi_{2k}(x)\,dx = \sqrt{2}\,a_0^{(k)}c_{2k}(0), \tag{2.18}$$

whence

$$P(0) = \pi\sqrt{2}\sum_{k=1}^{\infty} \alpha_{2k}a_0^{(k)}\left[\sqrt{2}\,a_0^{(k)}\gamma_0 - f_k\right], \quad P(\infty) = 2\pi\gamma_1. \tag{2.19}$$

Thus, according to (2.19) the quantity γ_0 (the initial introduction of the punch) is connected with the initial value of the indenting force, while its final value depends only on the velocity of the translatory motion of the punch γ_1. Exactly in the same way $M(\infty) = 2\pi/3\beta_1$, i.e., depends only on the angular velocity of rotation of the punch.

Following [10], with the expressions (2.8), (2.17)-(2.19) taken into account, we can state that the series (2.2) converges in $L_2(-1, 1)$ uniformly with respect to t on [0, T] for all $T > 0$ and determines, consequently, the generalized solution of the problem thus formulated in $L_2(-1, 1) \times C(0, T)$. Here C(0, T) is the space of continuous functions on [0, T].

Thus, the basic structure of the solution to Eq. (2.2) is (1.10).

3. The Case of Forces, Constant in Time, Pressing in the Punch. Let P = M = const. We assume here that the rigid-body motion of the punch, as a consequence of the wear of the surface of the region, varies with time according to the law

$$\gamma(t) = \gamma t + \sum_{k=0}^{\infty} \gamma_k e^{-\alpha_{2k}t}, \quad \beta(t) = \beta t + \sum_{k=0}^{\infty} \beta_k e^{-\alpha_{2k+1}t}, \tag{3.1}$$

where α_k, γ, β are constants, $\alpha_0 = \alpha_1 = 0$. Specification of $\gamma(t)$, $\beta(t)$ in the form (3.1) is justified by the fact that, as was shown in Sec. 2, for a sufficiently large t to certain constant values of P and M there corresponds linear wear.

According to Sec. 2, we shall seek the solution of Eq. (1.10) in the form

$$\varphi(x,t) = \sum_{k=0}^{\infty}\left[\varphi_{2k}(x)e^{-\alpha_{2k}t} + \varphi_{2k+1}(x)e^{-\alpha_{2k+1}t}\right] \quad (\alpha_i = \text{const}, \quad \alpha_0 = \alpha_1 = 0). \tag{3.2}$$

After substitution of (3.1), (3.2) into (1.10) and equating the coefficients of the left and right sides at t^0, t^1, $(1 - e^{-\alpha_{2k}t})$, $(1 - e^{-\alpha_{2k+1}t})$, we obtain

$$\varphi_0(x) = \pi\gamma, \quad \varphi_1(x) = \pi\beta x; \tag{3.3}$$

$$\alpha_{2k}\int_{-1}^{1}\varphi_{2k}(\xi)K\left(\frac{\xi - x}{\lambda}\right)d\xi = \pi\gamma_k\alpha_{2k} + \varphi_{2k}(x) \quad (|x| \leqslant 1, \ k \geqslant 1); \tag{3.4}$$

$$\alpha_{2k+1}\int_{-1}^{1}\varphi_{2k+1}(\xi)K\left(\frac{\xi - x}{\lambda}\right)d\xi = \pi\beta_k\alpha_{2k+1}x + \varphi_{2k+1}(x) \quad (|x| \leqslant 1, \ k \geqslant 1). \tag{3.5}$$

From (3.4), (3.5) it is seen that we can find the solution of the integral equation of Fredholm (in view of the lemma) of the second kind of the form

$$(E - \alpha B)\varphi + \pi\alpha g(x) = 0, \tag{3.6}$$

where the operator $B\varphi$ is given by the expression (2.7), while $g(x) = \gamma$ or $g(x) = \beta x$.

We note that in view of the properties of the operator $B\varphi$, which are indicated in Sec. 2, Eq. (3.6) for almost all α is uniquely solvable in $L_2(-1, 1)$. At the same time $\lambda \in (0, \infty)$.

We subsequently shall consider only the even case, bearing in mind, that for the odd case all is done analogously. We note that

$$P = \int_{-1}^{1}\varphi(\xi, t)\,d\xi = \sum_{k=0}^{\infty}P_k e^{-\alpha_{2k}t}; \tag{3.7}$$

$$P_0 = P = \int_{-1}^{1}\varphi_0(\xi)\,d\xi = 2\pi\gamma, \quad P_k = \int_{-1}^{1}\varphi_{2k}(\xi)\,d\xi = 0 \quad (k \geqslant 1). \tag{3.8}$$

We seek the solution of Eq. (3.4) in the form

$$\varphi_{2k}(x) = \pi\sqrt{2}\,\gamma_k\psi_{2k}(x), \quad \psi_{2k}(x) = \sum_{m=0}^{\infty}a_m^{(k)}P_{2m}^{*}(x). \tag{3.9}$$

Substituting (2.9), (3.9) into (3.4), using the orthogonality property of Legendre polynomials and equating in the resulting relation the coefficients of the right and left sides at polynomials of the same number, we obtain

$$\alpha_{2k}\sum_{m=0}^{\infty}a_m^{(k)}e_{nm}(\lambda) - a_n^{(k)} = \delta_{0n} \quad (n = 0, 1, 2, \ldots). \tag{3.10}$$

We note that in view of (3.8), (3.9)

$$P_k = \int_{-1}^{1}\varphi_{2k}(\xi)\,d\xi = 2\pi\gamma_k\alpha_{2k}a_0^{(k)} = 0, \quad a_0^{(k)} = 0 \quad (k \geqslant 1). \tag{3.11}$$

The condition (3.11) serves for the determination of the unknown quantities α_{2k}. Indeed, from the system (3.10) we have $a_0^{(k)} = \Delta_1/\Delta$, where Δ is the determinant of the system (3.10); Δ_1 is the determinant which is obtained from Δ by replacing in it the first column with the elements $\{1, 0, 0, \ldots, 0, \ldots\}$. The determinant Δ_1 is symmetrical; therefore its roots $\alpha = \alpha_{2k}$ ($k = 1, 2, \ldots$) are real. In addition, earlier it was mentioned that for sufficiently long time linear wear corresponds to the constant P and M; therefore the sums over the exponents in (3.1) must vanish for $t \to \infty$. Hence, in particular, it follows that $\alpha_i \geqslant 0$; in this we can convince ourselves directly by means of a subsequent computation criterion [11]. For each concrete problem we construct a collection of principal minors of the determinant Δ_1. From their nonnegativeness follows in fact nonnegativeness of the numbers α_{2k} ($k \geqslant 1$).

TABLE 1

ij	e_{ij} (1)	ij	e_{ij} (1)	ij	e_{ij} (1)
00	1,20168	11	0,74072	23	—0,12009
01	—0,16220	12	—0,17450	33	0,30896
02	—0,04062	13	—0,02811		
03	—0,00863	22	0,44640		

Having determined the numbers α_{2k}, we then find from the nonhomogeneous system (3.10) $a_m^{(k)}$(m = 1, 2, ...) and, in this way, construct the system of functions $\varphi_{2k}(x)$ ($k \geqslant 1$). We next find

$$\varphi(x, 0) = \sum_{k=0}^{\infty} \varphi_{2k}(x) = \pi \sqrt{2} \left[\sum_{k=1}^{\infty} \alpha_{2k}\gamma_k \sum_{m=1}^{\infty} a_m^{(k)} P_{2m}^*(x) + \gamma P_0^*(x) \right]. \tag{3.12}$$

Then the constants γ_k ($k \geqslant 1$) need be determined from the condition that Eq. (1.12) is satisfied for $\beta(0) \equiv 0$. Representing f(x) in the form $f(x) = \sum_{i=0}^{\infty} f_i P_{2i}^*(x)$, substituting (2.10), (3.12), and (1.12), we obtain

$$\sqrt{2} \sum_{k=1}^{\infty} \alpha_{2k}\gamma_k \sum_{m=1}^{\infty} a_m^{(k)} \sum_{j=0}^{\infty} e_{mj}(\lambda) P_{2j}^*(x) + \sqrt{2}\,\gamma \sum_{j=0}^{\infty} e_{0j}(\lambda) P_{2j}^*(x) = \sum_{m=0}^{\infty} \left[\sqrt{2}\,\gamma(0)\,\delta_{0m} - f_m\right] P_{2m}^*(x). \tag{3.13}$$

Having equated in the relation (3.13) the coefficients of the right and left sides at the Legendre polynomials of the same number, we have

$$\sum_{k=1}^{\infty} \alpha_{2k}\gamma_k \sum_{m=1}^{\infty} e_{mj}(\lambda) a_m^{(k)} + \gamma e_{0j}(\lambda) = [\gamma(0)\,\delta_{0j} - f_j 2^{-1/2}] \quad (j = 0, 1, 2, \ldots). \tag{3.14}$$

Here $\gamma(0)$ can be taken as not depending on γ_k ($k \geqslant 1$), for

$$\gamma(0) = \gamma_0 + \sum_{k=1}^{\infty} \gamma_k. \tag{3.15}$$

After solution of the system (3.14) the functions $\varphi_{2k}(x)$ will be completely determined and, consequently, the solution of the problem $\varphi(x, t)$ will be determined. Since in the course of solution of the infinite algebraic system (3.14) the constant γ is expressed in terms of $\gamma(0)$, the indenting force P can be connected with the initial entry $\gamma(0)$ of the punch into the layer.

4. Numerical Examples. In the role of an example we considered the plane contact problem with wear for an elastic isotropic layer of thickness h, with elastic constants (G – shear modulus, ν – Poisson's ratio), rigidly clamped along the base. Here $f(x) \equiv 0$, $\beta(t) \equiv 0$, $\lambda = h/a$ (a is the half-width of the punch), $\theta = G(1 - \nu)^{-1}$,

$$L(u) = \frac{2\sigma \operatorname{sh} 2u - 4u}{2\sigma \operatorname{ch} 2u + 1 + \sigma^2 + 4u^2} \quad (\sigma = 3 - 4\nu).$$

The coefficients $e_{ij}(\lambda)$, entering into the infinite system, are computed with accuracy up to $\epsilon = 10^{-5}$ for $\lambda = 1$, $\nu = 0.3$ and are recorded in Table 1.

In the case of linear variation with time of the rigid-body displacement of the punch, the infinite system (2.11) will be solved by the reduction method, confining the analysis to the first three equations. For the determination of α_{2k} we obtain an algebraic third-order equation, whence we find α_2, α_4, α_6. Accordingly we shall construct a system of functions $\varphi_{2k}(x)$ of the form (2.13), (2.14).

Determining the constants $c_{2k}(0)$ ($k \geqslant 1$) from (2.17) for $f_k \equiv 0$, we construct the solution of the problem according to the expressions (2.8), (2.2).

For the case of forces pressing in the punch which are constant in time, the infinite system (3.10) will also be solved by the reduction method. Having taken in (3.10) the four first equations, we find the unknown numbers α_2, α_4, α_6. Having constructed a system of functions $\varphi_{2k}(x)$ of the form (3.3), (3.9) containing four arbitrary constants γ, γ_1, γ_2, γ_3, we determine the latter subsequently from the truncated infinite system (3.14).

In Table 2 we have given the values of the numbers α_{2k} for the two problems, respectively, while in Table 3 we have given the values of the relative magnitude of the indenting force P(t)/P(0) for the first problem, and the relative magnitude of the translatory displacement of the punch for the second problem, computed according to the expressions

TABLE 2

k	1	2	3
α_{2k}	0,7979 1,2148	1,2758 2,1721	2,8433 4,7050

TABLE 3

	0	1	2	3	4	∞
$P(t)/P(0)$ $\gamma(t)/\gamma(0)$	0,99 1,000	0,41 1,916	0,18 2,794	0,08 3,669	0,03 4,543	0,00 ∞

TABLE 4

x	0	0,20	0,50	0,80	0,90	1,00
$\varphi(x,\ 1)/\gamma_0$ $\varphi(x,\ 1)/\gamma(0)$	1,084 2,604	1,108 2,607	1,228 2,687	1,126 2,927	1,016 2,977	0,833 2,929
$\varphi(x,\ 2)/\gamma_0$ $\varphi(x,\ 2)/\gamma(0)$	0,535 2,692	0,538 2,700	0,549 2,742	0,429 2,792	0,350 2,796	0,236 2,782
$\varphi(x,\ 4)/\gamma_0$ $\varphi(x,\ 4)/\gamma(0)$	0,121 2,739	0,118 2,740	0,107 2,744	0,0710 2,748	0,0553 2,748	0,0366 2,747

(2.18), (2.19), (3.1), (3.15). Here it was assumed that $P(\infty) = 0$ in the case of the first problem and $P(0) = 5.52\gamma_0$ [1].

In Table 4 we have presented the values of $\varphi(x,\ t)/\gamma_0$ and $\varphi(x,\ t)/\gamma(0)$ for the first and second problem respectively for various values of t. We note that for t = 0 for both cases the solutions are determined by expressions of the work [1], while for $t = \infty$ we have the relations

$$\varphi(x,\ \infty) = \pi\gamma'(\infty),\ P(\infty) = 2\pi\gamma'(\infty),$$

arising from the fact that $\gamma(t) = \gamma_0 + \gamma_1 t$, and also from the expressions (2.2), (3.1), (1.11) and $\varphi(x,\ \infty)/\gamma_0 = 0,\ \varphi(x,\ \infty)/\gamma(0) = 2.744$.

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